Oppgave 1

a) $3 \cdot 2 + 4$
$= 6 + 4$
$= \underline{10}$

b) $4 + 3 \cdot 2$
$= 4 + 6$
$= \underline{10}$

c) $3(-2) + 4$
$= -6 + 4$
$= \underline{-2}$

d) $4 + 3 \cdot (-2)$
$= 4 - 6$
$= \underline{-2}$

e) $2 \cdot 3^2 + 4$
$= 2 \cdot 9 + 4$
$= 18 + 4$
$= \underline{22}$

f) $2 \cdot (-3)^2 + 4$
$= 2 \cdot 9 + 4$
$= 18 + 4$
$= \underline{22}$

g) $(-3) \cdot (-2) - 7$
$= 6 - 7$
$= \underline{-1}$

h) $(-3) \cdot (-2)^3 - 7$
$= (-3) \cdot (-8) - 7$
$= 24 - 7$
$= \underline{17}$

i) $(2)^3 - 1$
$= 8 - 1$
$= \underline{7}$

j) $(-2)^3 - 1$
$= -8 - 1$
$= \underline{-9}$

k) $-2^3 - 1$
$= -8 - 1$
$= \underline{-9}$

l) $-(-2)^3 - 1$
$= -(-8) - 1$
$= 8 - 1$
$= \underline{7}$

Oppgave 2

a) $\frac{3}{6} = \frac{3 \cdot 1}{3 \cdot 2} = \frac{1}{2}$

b) $\frac{4}{24} = \frac{4 \cdot 1}{4 \cdot 6} = \frac{1}{6}$

c) $\frac{18}{27} = \frac{9 \cdot 2}{9 \cdot 3} = \frac{2}{3}$

d) $\frac{45}{165} = \frac{5 \cdot 9}{5 \cdot 33} = \frac{3 \cdot 3}{3 \cdot 11} = \frac{3}{11}$

e) $\frac{315}{420} = \frac{5 \cdot 3 \cdot 3 \cdot 7}{5 \cdot 3 \cdot 2 \cdot 2 \cdot 7} = \frac{3}{4}$

Oppgave 3

a) $\frac{1}{6} + \frac{2}{3}$

$= \frac{1}{6} + \frac{2 \cdot 2}{3 \cdot 2}$

$= \frac{1}{6} + \frac{4}{6}$

$= \frac{1+4}{6}$

$= \frac{5}{6}$

b) $\frac{3}{4} - \frac{3}{8}$

$= \frac{3 \cdot 2}{4 \cdot 2} - \frac{3}{8}$

$= \frac{6}{8} - \frac{3}{8}$

$= \frac{6-3}{8}$

$= \frac{3}{8}$

Oppg 3.

c) $3 + \frac{5}{6}$

$= \frac{3 \cdot 6}{6} + \frac{5}{6}$

$= \frac{18}{6} + \frac{5}{6}$

$= \frac{18+5}{6}$

$= \frac{23}{6}$

d) $\frac{1}{4} + \frac{2}{3}$

$= \frac{1 \cdot 3}{4 \cdot 3} + \frac{2 \cdot 4}{3 \cdot 4}$

$= \frac{3}{12} + \frac{8}{12}$

$= \frac{3+8}{12}$

$= \frac{11}{12}$

e) $\frac{5}{6} \cdot \frac{2}{15}$

$= \frac{5 \cdot 2 \cdot 1}{2 \cdot 3 \cdot 5 \cdot 3}$

$= \frac{1}{9}$

f) $5 \cdot \frac{2}{3}$

$= \frac{5}{1} \cdot \frac{2}{3}$

$= \frac{5 \cdot 2}{1 \cdot 3}$

$= \frac{10}{3}$

eller $5 \cdot \frac{2}{3}$

$= \frac{5 \cdot 2}{3}$

$= \frac{10}{3}$

g) $\frac{1}{12} : \frac{2}{3}$

$= \frac{1}{12} \cdot \frac{3}{2}$

$= \frac{1 \cdot 3}{3 \cdot 4 \cdot 2}$

$= \frac{1}{8}$

h) $5 : \frac{2}{3}$

$= \frac{5}{1} \cdot \frac{3}{2}$

$= \frac{5 \cdot 3}{1 \cdot 2}$

$= \frac{15}{2}$

Oppgave 4

a) $2\left(\frac{1}{4}-\frac{3}{8}\right)$

$= 2\cdot\left(\frac{1\cdot 2}{4\cdot 2}-\frac{3}{8}\right)$

$= 2\cdot\left(\frac{2}{8}-\frac{3}{8}\right)$

$= 2\cdot\frac{(2-3)}{8}$

$= 2\cdot\left(-\frac{1}{8}\right)$

$= \frac{\cancel{2}^{1}\cdot(-1)}{8_{4}}$

$= \frac{-1}{4} = -\frac{1}{4}$

b) $\left(\frac{3}{4}-\frac{1}{5}\right)\cdot\frac{5}{4}$

$= \left(\frac{3\cdot 5}{4\cdot 5}-\frac{1\cdot 4}{5\cdot 4}\right)\cdot\frac{5}{4}$

$= \left(\frac{15}{20}-\frac{4}{20}\right)\cdot\frac{5}{4}$

$= \left(\frac{15-4}{20}\right)\cdot\frac{5}{4}$

$= \frac{11}{20}\cdot\frac{5}{4}$

$= \frac{11\cdot\cancel{5}}{4\cdot\cancel{5}\cdot 4}$

$= \frac{11}{16}$

c) $\left(1-\frac{5}{6}\right)\left(\frac{5}{6}+\frac{1}{3}\right)$

$= \left(\frac{6}{6}-\frac{5}{6}\right)\cdot\left(\frac{5}{6}+\frac{1\cdot 2}{3\cdot 2}\right)$

$= \left(\frac{6-5}{6}\right)\cdot\left(\frac{5}{6}+\frac{2}{6}\right)$

$= \frac{1}{6}\cdot\left(\frac{5+2}{6}\right)$

$= \frac{1}{6}\cdot\frac{7}{6}$

$= \frac{1\cdot 7}{6\cdot 6} = \frac{7}{36}$

d) $\frac{\frac{1}{2}}{\frac{1}{5}} = \frac{1}{2}:\frac{1}{5}$

$= \frac{1}{2}\cdot\frac{5}{1}$

$= \frac{1\cdot 5}{2\cdot 1}$

$= \frac{5}{2}$

Oppg. 4

e) $\dfrac{\frac{21}{36}}{\frac{14}{45}} = \frac{21}{36} : \frac{14}{45} = \frac{21}{36} \cdot \frac{45}{14}$

$= \dfrac{3 \cdot 7 \cdot 3 \cdot 5 \cdot 3}{3 \cdot 3 \cdot 2 \cdot 2 \cdot 2 \cdot 7} = \frac{5 \cdot 3}{2 \cdot 2 \cdot 2} = \underline{\underline{\frac{15}{8}}}$

f) $\dfrac{\frac{1}{2} + \frac{3}{4}}{\frac{1}{5} + 3} = \dfrac{\frac{1 \cdot 2}{2 \cdot 2} + \frac{3}{4}}{\frac{1}{5} + \frac{3 \cdot 5}{5}} = \dfrac{\frac{2}{4} + \frac{3}{4}}{\frac{1}{5} + \frac{15}{5}}$

$= \dfrac{\frac{2+3}{4}}{\frac{1+15}{5}} = \dfrac{\frac{5}{4}}{\frac{16}{5}} = \frac{5}{4} : \frac{16}{5} = \frac{5}{4} \cdot \frac{5}{16}$

$= \underline{\underline{\frac{25}{64}}}$

g) $\dfrac{2 - \frac{1}{2}}{\frac{1}{5} \cdot \frac{5}{6}} + \frac{3}{5} = \dfrac{\frac{2 \cdot 2}{2} - \frac{1}{2}}{\frac{1 \cdot 5}{5 \cdot 6}} + \frac{3}{5}$

$= \dfrac{\frac{4-1}{2}}{\frac{1}{6}} + \frac{3}{5} = \dfrac{\frac{3}{2}}{\frac{1}{6}} + \frac{3}{5} = \left(\frac{3}{2} : \frac{1}{6}\right) + \frac{3}{5}$

$= \frac{3}{2} \cdot \frac{6^3}{1} + \frac{3}{5} = 9 + \frac{3}{5} = \frac{9 \cdot 5}{5} + \frac{3}{5} = \frac{45+3}{5} = \underline{\underline{\frac{48}{5}}}$

Oppgave 5

a) $3a + 5a - b = \underline{8a - b}$

b) $3a + 4a - b + 5b - 7a = \underline{4b}$

c) $6a + 2(5a + b) - 3b$

$= 6a + 10a + 2b - 3b$

$= \underline{16a - b}$

d) $a^2 + 4b - 5a^2 - 5a + 8a^2 + b$

$= \underline{4a^2 - 5a + 5b}$

e) $xy + 5x - xy + x^2y - x^2$

$= 5x + x^2y - x^2$

$= \underline{-x^2 + x^2y - 5x}$

f) $3a + 5a - b$

$= \underline{8a - b}$

g) $3(ab + b) + 2(a - b)$

$= 3ab + 3b + 2a - 2b$

$= \underline{3ab + 2a + b}$

Oppg 5

h) $3(ab-b)-2(a-b)$

$= 3ab - 3b - 2a + 2b$

$= \underline{3ab - 2a - b}$

i) $-(ab+b)-(a-b)$

$-ab - b - a + b$

$= \underline{-ab - a}$

(eller
$= -1\cdot(ab+b) - 1\cdot(a-b)$
$= \ldots$ osv)

j) $3(ab+b)-(a-b)2$

$= 3(ab+b) - 2(a-b)$

$= 3ab + 3b - 2a + 2b$

$= \underline{3ab - 2a + 5b}$

Oppgave 6

a) $\frac{x}{6} + \frac{x}{3} = \frac{x}{6} + \frac{x\cdot 2}{3\cdot 2} = \frac{x}{6} + \frac{2x}{6}$

$= \frac{x+2x}{6} = \frac{3x}{6_2} = \frac{x}{2} = \underline{\frac{1}{2}x}$

opp 6

b) $\frac{3}{4a} - \frac{3}{8a} = \frac{3 \cdot 2}{4a \cdot 2} - \frac{3}{8a}$

$= \frac{6}{8a} - \frac{3}{8a} = \frac{6-3}{8a} = \frac{3}{8a}$

c) $\frac{3}{x} + \frac{1}{2x} + \frac{2}{3x}$

$= \frac{6 \cdot 3}{6 \cdot x} + \frac{3 \cdot 1}{3 \cdot 2x} + \frac{2 \cdot 2}{2 \cdot 3x}$

$= \frac{18}{6x} + \frac{3}{6x} + \frac{4}{6x}$

$= \frac{18 + 3 + 4}{6x}$

$= \frac{25}{6x}$

d) $\frac{9a}{4} \cdot \frac{2}{3a}$

$= \frac{3 \cdot 3 \cdot a \cdot 2}{2 \cdot 2 \cdot 3 \cdot a}$

$= \frac{3}{2}$

oppg 6

e) $\frac{5x^2}{6y} \cdot \frac{3y^2}{15x}$

$= \frac{5 \cdot x \cdot x \cdot 3y \cdot y}{2 \cdot 3y \cdot 3 \cdot 5x}$

$= \frac{xy}{2 \cdot 3} = \frac{xy}{6} = \frac{1}{6}xy$

f) $\frac{5a}{6} : \frac{15a}{3}$

$= \frac{5a}{6} \cdot \frac{3}{15a}$

$= \frac{5a \cdot 3 \cdot 1}{2 \cdot 3 \cdot 3 \cdot 5a}$

$= \frac{1}{6}$

g) $\frac{x}{12} : 3x$

$= \frac{x}{12} : \frac{3x}{1}$

$= \frac{x}{12} \cdot \frac{1}{3x}$

$= \frac{x \cdot 1}{12 \cdot 3x} = \frac{1}{36}$

Oppgave 7

10/30

a) $2x = 18 \quad | :2$

$\frac{2x}{2} = \frac{18}{2}$

$\underline{x = 9}$

b) $\frac{2}{3}x = 10 \quad | \cdot \frac{3}{2}$

$\frac{2}{3}x \cdot \frac{3}{2} = 10 \cdot \frac{3}{2}$

$x = \frac{30}{2} = \underline{15}$

c) $3x = \frac{1}{6} \quad | :3$

$\frac{3x}{3} = \frac{1}{6} : \frac{3}{1}$

$x = \frac{1}{6} \cdot \frac{1}{3}$

$x = \frac{1}{6 \cdot 3}$

$\underline{x = \frac{1}{18}}$

<u>eller:</u> $3x = \frac{1}{6}$

$x = \frac{1}{6 \cdot 3}$

Du ser at 3-tallet havner under brøkstreken.

d) $\frac{1}{4}x = \frac{3}{2} \quad | \cdot 4$

$4 \cdot \frac{1}{4}x = 4 \cdot \frac{3}{2}$

$x = \frac{4 \cdot 3}{2} = \frac{12}{2} = \underline{6}$

e) $\frac{3}{5} = \frac{3x}{2}$

Kryssmultipliserer

$3 \cdot 2 = 5 \cdot 3x$

$6 = 15x \quad | :15$

$\frac{6}{15} = x$ Speil vender

$x = \frac{6}{15} = \frac{2}{5}$

$\underline{x = \frac{2}{5}}$

f) $2x - 3 = 11$

$2x = 11 + 3$

$2x = 14 \quad | :2$

$x = \frac{14}{2}$

$\underline{x = 7}$

Oppg 7

g) $3a+4=a+12$

$3a-a=12-4$

$2a=8$

$a=\frac{8}{2}$

$a=4$

h) $x-3=-3x-7$

$x+3x=-7+3$

$4x=-4$

$x=\frac{-4}{4}$

$x=-1$

i) $11a=2(5a+1)-3a$

$11a=10a+2-3a$

$11a-10a+3a=2$

$4a=2$

$a=\frac{2}{4}$

$a=\frac{1}{2}$

j) $2(x-4)=3(5-2x)$

$2x-8=15-6x$

$2x+6x=15+8$

$8x=23$

$x=\frac{23}{8}$

k)

$0,01(x+45)=0,003(x-30)$

ganger begge sider med 1000

$10(x+45)=3(x-30)$

$10x+450=3x-90$

$10x-3x=-90-450$

$7x=-540$

$x=-\frac{540}{7}$

(77 hele og $\frac{1}{7}$)

k)

$0,01(x+45)=0,003(x-30)$

$0,01x+0,45=0,003x-0,09$

$0,01x-0,003x=-0,09-0,45$

$0,007x=-0,54$

$x=\frac{-0,54}{0,007}$

$x=-77,142$

$x\approx -77,14$

eller se ovenfor!

Oppgave 8

a) $\frac{5}{6}x + 2 = \frac{1}{3}x - \frac{1}{6} \quad | \cdot 6$

$\frac{5x \cdot 6}{6} + 2 \cdot 6 = \frac{1 \cdot x \cdot 6}{3} - \frac{1 \cdot 6}{6}$

$5x + 12 = 2x - 1$

$5x - 2x = -1 - 12$

$3x = -13$

$x = \frac{-13}{3}$

b) $\frac{3}{x} - 2 = \frac{5}{x}$ (x kan ikke være null!)

$| \cdot x$

$\frac{3x}{x} - 2x = \frac{5x}{x}$

$3 - 2x = 5$

$-2x = 5 - 3$

$x = \frac{2}{-2}$

$x = -1$

c) $\frac{(x-1)}{3} = \frac{(2x+1)}{4}$

$4(x-1) = 3(2x+1)$

$4x - 4 = 6x + 3$

$4x - 6x = 3 + 4$

$-2x = 7$

$x = -\frac{7}{2}$

d) $\frac{(x-1)}{3} + 1 = \frac{(2x+1)}{4} \quad | \cdot 12$

$\frac{12(x-1)}{3} + 12 = \frac{12(2x+1)}{4}$

$4(x-1) + 12 = 3(2x+1)$

$4x - 4 + 12 = 6x + 3$

$4x - 6x = 3 + 4 - 12$

$-2x = -5$

$x = \frac{-5}{-2}$

$x = \frac{5}{2}$

Oppgave 9

a) $m = \frac{G}{g}$, $g = \frac{G}{m}$

b) $k = \frac{F}{x}$, $x = \frac{F}{k}$

c) $m = \frac{Q}{s}$, $s = \frac{Q}{m}$

d) $m = \frac{C}{c}$, $c = \frac{C}{m}$

e) $P = \frac{W}{t}$, $t = \frac{W}{P}$

f) $s = v \cdot t$, $t = \frac{s}{v}$

g) $m = \rho \cdot V$, $V = \frac{m}{\rho}$

h) $R = \frac{U^2}{P}$, $U^2 = P \cdot R$

$\underline{U = \sqrt{P \cdot R}}$

i) $m = \frac{Ep}{gh}$, $g = \frac{Ep}{mh}$, $h = \frac{Ep}{mg}$

j) $Ek = \frac{1}{2} m v^2$ 1.2

$2Ek = m v^2$

$m = \frac{2Ek}{v^2}$

$v^2 = \frac{2Ek}{m}$

$\underline{v = \sqrt{\frac{2Ek}{m}}}$

9)

k) $E_p = \frac{1}{2} k x^2 \quad | \cdot 2$

$2E_p = kx^2$

$k = \frac{2E_p}{x^2}$

$x^2 = \frac{2E_p}{k}$

$x = \sqrt{\frac{2E_p}{k}}$

l) $F = \frac{W}{s \cdot \cos\alpha}$, $s = \frac{W}{F \cdot \cos\alpha}$, $\cos\alpha = \frac{W}{F \cdot s}$

m) $p_1 = \frac{p_2 \cdot V_2 \cdot T_1}{T_2 \cdot V_1}$, $V_1 = \frac{p_2 \cdot V_2 \cdot T_1}{T_2 \cdot p_1}$

$T_1 = \frac{p_1 \cdot V_1 \cdot T_2}{p_2 \cdot V_2}$, $p_2 = \frac{p_1 \cdot V_1 \cdot T_2}{T_1 \cdot V_2}$

$V_2 = \frac{p_1 \cdot V_1 \cdot T_1}{T_1 \cdot p_2}$, $T_2 = \frac{p_2 \cdot V_2 \cdot T_1}{p_1 \cdot V_1}$

n) $P_{avgitt} = \eta \cdot P_{tilfort}$, $P_{tilfort} = \frac{P_{avgitt}}{\eta}$

o) $m = \frac{Q}{c \cdot \Delta T}$, $c = \frac{Q}{m \cdot \Delta T}$, $\Delta T = \frac{Q}{c \cdot m}$

p) $\overline{E_k} = \frac{3}{2} kT \quad | \cdot 2$

$2\overline{E_k} = 3kT$

$k = \frac{2\overline{E_k}}{3T}$, $T = \frac{2\overline{E_k}}{3k}$

p) eller

$k = \frac{\overline{E_k}}{\frac{3}{2}T}$

og $T = \frac{\overline{E_k}}{\frac{3}{2}k}$

Oppgave 10

a) $E_k = E_{total} - E_p$, $E_p = E_{total} - E_k$

b) $E_{p1} = E_{p2} + E_{k2} - E_{k1}$

$E_{k1} = E_{p2} + E_{k2} - E_{p1}$

$E_{p2} = E_{p1} + E_{k1} - E_{k2}$

$E_{k2} = E_{p1} + E_{k1} - E_{p2}$

c) $U = U_1 + R \cdot I$

$U - R_1 \cdot I = U_1$

$U_1 = U - R_1 \cdot I$

$U = U_1 + R \cdot I$

$U - U_1 = R \cdot I$

$R = \frac{U - U_1}{I}$

$I = \frac{U - U_1}{R}$

d) $p = p_0 + \rho \cdot g h$

$p_0 = p - \rho \cdot g \cdot h$

$p = p_0 + \rho g h$

$p - p_0 = \rho g h$

$\rho = \frac{p - p_0}{g h}$

$g = \frac{p - p_0}{\rho h}$

$h = \frac{p - p_0}{\rho g}$

e) $v_0 = v - at$

$a = \frac{v - v_0}{t}$, $t = \frac{v - v_0}{a}$

f) $c = \frac{Q_m}{m(t_2 - t_1)}$

$m = \frac{Q_m}{c(t_2 - t_1)}$

$Q_m = c \cdot m (t_2 - t_1)$

$\frac{Q_m}{c \cdot m} = t_2 - t_1$

$t_2 = \frac{Q_m}{c \cdot m} + t_1$

$t_1 = t_2 - \frac{Q_m}{cm}$

10/ g) $s = \frac{(v_0 + v)}{2} \cdot t$

$2s = (v_0 + v) \cdot t$

$t = \frac{2s}{v_0 + v}$

$\frac{2s}{t} = v_0 + v$

$v_0 = \frac{2s}{t} - v$

$v = \frac{2s}{t} - v_0$

h) $v^2 - v_0^2 = 2as$

$v^2 = 2as + v_0^2$

$v = \sqrt{2as + v_0^2}$

$v_0 = \sqrt{v^2 - 2as}$

$a = \frac{v^2 - v_0^2}{2s}$

$s = \frac{v^2 - v_0^2}{2a}$

i) $s = v_0 t + \frac{1}{2} a t^2$ / 1.2

$2s = 2v_0 t + at^2$

$2s - at^2 = 2v_0 t$

$v_0 = \frac{2s - at^2}{2t}$

$2s - 2v_0 t = at^2$

$a = \frac{2s - 2v_0 t}{t^2}$

j) $R_2 = R_1(1 + \alpha \cdot \Delta t)$

$R_1 = \frac{R_2}{1 + \alpha \cdot \Delta t}$

$R_2 = R_1(1 + \alpha \Delta t)$

$R_2 = R_1 + R_1 \alpha \cdot \Delta t$

$R_2 - R_1 = R_1 \alpha \cdot \Delta t$

$\Delta t = \frac{R_2 - R_1}{R_1 \alpha}$

$\alpha = \frac{R_2 - R_1}{R_1 \cdot \Delta t}$

10/ k) $R_2 = R_1(1+\alpha(t_2-t_1))$

$$R_1 = \frac{R_2}{1+\alpha(t_2-t_1)}$$

$$R_2 = R_1(1+\alpha(t_2-t_1))$$

$$R_2 = R_1 + R_1\alpha(t_2-t_1)$$

$$R_2 - R_1 = R_1\alpha(t_2-t_1)$$

$$\alpha = \frac{R_2-R_1}{R_1(t_2-t_1)}$$

$$\frac{R_2-R_1}{R_1\alpha} = t_2 - t_1$$

$$t_2 = \frac{R_2-R_1}{R_1\cdot\alpha} + t_1$$

$$t_1 = t_2 - \frac{R_2-R_1}{R_1\cdot\alpha}$$

eller

$$R_2 - R_1 = R_1\alpha\cdot t_2 - R_1\alpha t_1$$

$t_1 = ?$ $\quad R_1\alpha t_1 = R_1\alpha t_2 - R_2 + R_1$

$$t_1 = \frac{R_1\alpha t_2 - R_2 + R_1}{R_1\alpha}$$

$t_2 = ?$ $\quad R_2 - R_1 + R_1\alpha t_1 = R_1\alpha t_2$

$$t_2 = \frac{R_2 - R_1 + R_1\alpha t_1}{R_1\cdot\alpha}$$

10/ l) $W = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$ 1.2

$2W = mv^2 - mv_0^2$

$2W + mv_0^2 = mv^2$

$v^2 = \frac{2W + mv_0^2}{m}$

$v = \sqrt{\frac{2W + mv_0^2}{m}}$

↳ $mv_0^2 = mv^2 - 2W$

$v_0 = \sqrt{\frac{mv^2 - 2W}{m}}$

$m = ?$ $W = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$ |·2

$2W = mv^2 - mv_0^2$

$2W = m(v^2 - v_0^2)$

$m = \frac{2W}{v^2 - v_0^2}$

PROBLEM:
m er i 2 ledd!
En eneste mulighet for å isolere den er å sette på utsiden av en parentes.
Dette gjelder alltid når du har den ukjente i flere ledd

Oppgave 11

a) $\mu = \frac{F_f}{F_N}$, $F_N = \frac{F_f}{\mu}$

b) $W_i = \frac{P_i}{n_a \cdot i}$, $n_a = \frac{P_i}{W \cdot i}$, $i = \frac{P_i}{W_i \cdot n_a}$

c) $P_e = n_m \cdot P_i$, $P_i = \frac{P_e}{n_m}$

d) $A_M = C_M \cdot B \cdot T$, $B = \frac{A_M}{C_M \cdot T}$, $T = \frac{A_M}{C_M \cdot B}$

e) $k_1 = \frac{F}{v^2}$, $v^2 = \frac{F}{k_1}$

$\Downarrow$

$v = \sqrt{\frac{F}{k_1}}$

f) $V_h = \frac{\pi D^2}{4} \cdot S \quad \cdot 4$

$4V_h = \pi D^2 \cdot S$

$S = \frac{4V_h}{\pi D^2}$

$D^2 = \frac{4V_h}{\pi S}$

$D = \sqrt{\frac{4V_h}{\pi S}}$

$\left(D = 2\sqrt{\frac{V_h}{\pi S}} \right)$

g) $C_1 = \frac{BM \cdot C_B \cdot T}{B^2}$

$C_B = \frac{C_1 \cdot B^2}{BM \cdot T}$

$T = \frac{C_1 \cdot B^2}{BM \cdot C_B}$

$B^2 = \frac{BM \cdot C_B \cdot T}{C_1}$

$B = \sqrt{\frac{BM \cdot C_B \cdot T}{C_1}}$

(11) h) $k_i = \frac{P_t}{F \cdot v^3}$, $F = \frac{P_t}{k_i \cdot v^3}$

$$v^3 = \frac{P_t}{k_i \cdot F} \Rightarrow v = \sqrt[3]{\frac{P_t}{k_i \cdot F}}$$

i) $M = \frac{F_p \cdot R}{v^2}$, $v = \sqrt{\frac{F_p \cdot R}{M}}$, $R = \frac{M v^2}{F_p}$

j) $S = \frac{v \cdot n}{p} \cdot \left(\frac{d}{C}\right)^2 \Rightarrow v = \frac{S \cdot p}{n \cdot \left(\frac{d}{C}\right)^2}$

eller vi kan løse opp parantesen

$$S = \frac{v \cdot n \cdot d^2}{p \cdot C^2}$$ } og nå er det lettere å se hva som må gjøres

$$V = \frac{S \cdot p \cdot C^2}{n \cdot d^2} \qquad d^2 = \frac{S \cdot p \cdot C^2}{v \cdot n}$$

$$n = \frac{S \cdot p \cdot C^2}{v \cdot d^2} \qquad \hookrightarrow d = \sqrt{\frac{S \cdot p \cdot C^2}{v \cdot n}}$$

$$p = \frac{v \cdot n \cdot d^2}{S \cdot C^2}$$

eller $d = C \cdot \sqrt{\frac{S \cdot p}{v \cdot n}}$

$$C = \sqrt{\frac{v \cdot n \cdot d^2}{S \cdot p}}$$

eller $C = d \sqrt{\frac{v \cdot n}{S \cdot p}}$

(11) k) $fe = \frac{1}{2\pi} \cdot \sqrt{\frac{k}{m}}$

$2\pi \cdot fe = \sqrt{\frac{k}{m}}$

$(2\pi fe)^2 = \left(\sqrt{\frac{k}{m}}\right)^2$

$(2\pi fe)^2 = \frac{k}{m}$

$\rightarrow \underline{k = m \cdot (2\pi fe)^2}$

$\underline{m = \frac{k}{(2\pi fe)^2}}$

l) $T = \frac{2\pi R}{\sqrt{(g \cdot GM)}} \rightarrow \underline{R = \frac{T \cdot \sqrt{(g \cdot GM)}}{2\pi}}$

$T \cdot \sqrt{(g \cdot GM)} = 2\pi R$

$\sqrt{g \cdot GM} = \frac{2\pi R}{T}$ $(\ )^2$

$g \cdot GM = \left(\frac{2\pi R}{T}\right)^2 = \frac{(2\pi R)^2}{T^2}$

$\underline{GM = \frac{(2\pi R)^2}{g \cdot T^2}}$ oder $\underline{GM = \frac{4\pi^2 R^2}{g \cdot T^2}}$

Oppgave 12

a) $KB = k_1 \cdot T + BM$, $k_1 = ?$

$KB - BM = k_1 \cdot T$

$k_1 = \frac{KB - BM}{T}$ $\qquad$ $T = \frac{KB - BM}{k_1}$

$BM = KB - k_1 \cdot T$

b) $k = \frac{Q}{A(t_1 - t_2)}$, $A = \frac{Q}{k(t_1 - t_2)}$

$Q = k \cdot A(t_1 - t_2)$

$\frac{Q}{k \cdot A} = t_1 - t_2$

$t_1 = \frac{Q}{k \cdot A} + t_2$

$t_2 = t_1 - \frac{Q}{kA}$

c) $KB = T - \frac{1}{3}\left(\frac{T}{2} + \frac{\nabla}{A}\right)$

$KB = T - \frac{T}{6} - \frac{\nabla}{3A}$ $\qquad$ $T = ?$

$KB + \frac{\nabla}{3A} = T - \frac{T}{6}$

$1T - \frac{1}{6}T$

$KB + \frac{\nabla}{3A} = \frac{5}{6}T \quad | \cdot 6$

$6KB + \frac{6\nabla}{3A} = 5T \quad | : 5$

$T = \frac{6KB + \frac{2\nabla}{A}}{5}$

$T = \frac{6}{5}KB + \frac{2\nabla}{5A}$

(12) c) fortsettelse fra

$$KB + \frac{\nabla}{3A} = \frac{5T}{6} \quad , \nabla = ?$$

$$\frac{\nabla}{3A} = \frac{5T}{6} - KB \qquad | \cdot 3A$$

$$\nabla = 3A\left(\frac{5}{6}T - KB\right)$$

(eller $\nabla = \frac{5}{2}A \cdot T - 3A \cdot KB$)

$$A = \frac{\nabla}{3\left(\frac{5}{6}T - KB\right)}$$

(kan pyntes på ...)

lettere å gjøre det penere herfra:

$$\frac{\nabla}{3A} = \frac{5T}{6} - KB \qquad | \cdot 6A$$

$$\frac{2\cancel{6}A\nabla}{\cancel{3}A} = \frac{\cancel{6}A \cdot 5T}{\cancel{6}} - 6 \cdot A \cdot KB$$

$$2\nabla = 5AT - 6A \cdot KB$$

men nå fikk vi også A i flere ledd!

$$2\nabla = A(5T - 6KB)$$

$$A = \frac{2\nabla}{5T - 6KB}$$

(12)

d) $T = KB(1 + C_{pr})$

$$KB = \frac{T}{(1+C_{pr})}$$

$$1 + C_{pr} = \frac{T}{KB} \Rightarrow C_{pr} = \frac{T}{KB} - 1$$

e) $D = \frac{N-M}{R}$, $N = RD + M$, $M = N - RD$

f) $E = G \cdot 2(1+\mu)$

$E = 2G(1+\mu)$

g) $d = 2\sqrt{h(2r-h)}$ (r = ?)

$$\frac{d}{2} = \sqrt{h(2r-h)} \quad | \,(\,)^2$$

$$\left(\frac{d}{2}\right)^2 = h(2r-h)$$

$$\frac{d^2}{4} = h(2r-h) \quad | : h$$

$$\frac{d^2}{4h} = 2r - h$$

$$\frac{d^2}{4h} + h = 2r$$

$$r = \frac{\frac{d^2}{4h} + h}{2}$$

eller: $d^2 = 4h \cdot 2r - 4h^2$

$d^2 = 8hr - 4h^2$

$d^2 + 4h^2 = 8hr$

$$r = \frac{d^2 + 4h^2}{8h}$$

(12) h) $S = 4\pi R\sqrt{\frac{r^2+R^2}{2}}$

$\frac{S}{4\pi R} = \sqrt{\frac{r^2+R^2}{2}}$ $\quad | (\,)^2$

$\left(\frac{S}{4\pi R}\right)^2 = \frac{r^2+R^2}{2}$

$\frac{S^2}{16\pi^2 R^2} = \frac{r^2+R^2}{2}$ $\quad | \cdot 2$

$\frac{2S^2}{16\pi^2 R^2} = r^2+R^2$

$\frac{S^2}{8\pi^2 R^2} - R^2 = r^2 \Longrightarrow r = \sqrt{\frac{S^2}{8\pi^2 R^2} - R^2}$

eller $r = \sqrt{\frac{S^2 - R^4}{8\pi^2 R^2}}$

$R = 2$

$\frac{S^2}{8\pi^2 R^2} = r^2 + R^2$ $\quad | \cdot R^2$

$\frac{S^2}{8\pi^2} = R^2 \cdot r^2 + R^4$

$R^4 + r^2 \cdot R^2 - \frac{S^2}{8\pi^2} = 0$

Setter $R^2 = x$

$x^2 + r^2 \cdot x - \frac{S^2}{8\pi^2} = 0$

$ax^2 + bx + c = 0$

kan settes inn i løsningsformelen

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

hvor $a = 1$, $b = r^2$ og $c = -\frac{S^2}{8\pi^2}$

Dette ble vel aldri mye...

(12) i) Denne er også krevende

$$T = 4\pi\sqrt{(\quad)}$$

$$\frac{T}{4\pi} = \sqrt{(\quad)} \quad | (\,)^2$$

$$\left(\frac{T}{4\pi}\right)^2 = \frac{(M+3m)L}{3(M+2m)g} \Rightarrow L = \left(\frac{T}{4\pi}\right)^2 \cdot \frac{3(M+2m)g}{(M+3m)}$$

kan pyntes litt ...

For å finne M og m, settes gjerne $\left(\frac{T}{4\pi}\right)^2 = x$ for å få et penere uttrykk underveis. (bare som en hjelp)

$$x = \frac{(M+3m)L}{3(M+2m)g}$$

$$x \cdot 3(M+2m)g = (M+3m)L$$

$$3xMg + 6xmg = ML + 3mL$$

$$3xMg - ML = 3mL - 6xmg$$

$$M(3xg - L) = 3mL - 6xmg$$

$$M = \frac{3mL - 6xmg}{3xg - L}$$

x må settes inn!

$$M = \frac{3mL - 6 \cdot \left(\frac{T}{4\pi}\right)^2 \cdot mg}{3 \cdot \left(\frac{T}{4\pi}\right)^2 g - L}$$

Kan pyntes på.

(12) KEEVENDE!?!? OBS 27/30

i) $m = ?$

Vi har $\left(\frac{T}{4\pi}\right)^2 = \frac{(M+3m)L}{3(M+2m)g}$

$$\frac{T^2}{16\pi^2} = \frac{ML+3mL}{3Mg+6mg}$$

$$\frac{T^2}{16\pi^2}(3Mg+6mg) = ML+3mL$$

$$\frac{T^2 \cdot 3Mg}{16\pi^2} + \frac{T^2 \cdot 6mg}{16\pi^2} = ML+3mL$$

$$\frac{T^2 \cdot 3mg}{8\pi^2} - 3mL = ML - \frac{3T^2Mg}{16\pi^2}$$

$$m\left(\frac{T^2 \cdot 3g}{8\pi^2} - 3L\right) = ML - \frac{3T^2Mg}{16\pi^2}$$

$$m = \frac{ML - \frac{3T^2 \cdot Mg}{16\pi^2}}{\frac{T^2 \cdot 3g}{8\pi^2} - 3L}$$ } Kan gynta på ...

$$m = \frac{16\pi^2ML - 3T^2Mg}{6T^2g - 48\pi^2L}$$

(12)

j) $d = \sqrt[3]{\frac{P}{Q-P}}$ $\qquad (\quad)^3$

$$d^3 = \frac{P}{Q-P}$$

$$d^3(Q-P) = P$$

$$Q-P = \frac{P}{d^3}$$

$$\rightarrow \quad Q = \frac{P}{d^3} + P$$

P = ?

$$Q = \frac{P}{d^3} + P$$

$$Q = P\left(\frac{1}{d^3} + 1\right) \quad / \quad P = \frac{Q}{\frac{1}{d^3} + 1}$$

$$P = \frac{d^3 Q}{1 + d^3}$$

k)

$$n_e = \frac{Q_t - Q_v}{Q_t}$$

$$n_e Q_t = Q_t - Q_v \quad \rightarrow \quad Q_v = Q_t - n_e Q_t$$

$$Q_v = Q_t - n_e Q_t$$

$$Q_v = Q_t(1 - n_e) \quad \rightarrow \quad Q_t = \frac{Q_v}{1 - n_e}$$

(12) l) $T - W = \frac{Wv^2}{gx} \Rightarrow T = \frac{Wv^2}{gx} + W$

$gx(T-W) = Wv^2$

$\frac{gx(T-W)}{W} = v^2 \Rightarrow v = \sqrt{\frac{gx(T-W)}{W}}$

$W = ?$

$gx(T-W) = Wv^2$

$gxT - gxW = Wv^2$

$gxT = Wv^2 + gxW$

$gxT = W(v^2 + gx) \Rightarrow W = \frac{gxT}{v^2 + gx}$

m) $\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$ | $(\,)^2$

$\omega^2 = \frac{1}{LC} - \frac{R^2}{4L^2}$ | $\cdot 4L^2C$

$4L^2C\omega^2 = \frac{4L^2C}{LC} - \frac{4L^2C \cdot R^2}{4L^2}$

$4L^2C\omega^2 = 4L - CR^2$

$CR^2 = 4L - 4L^2C\omega^2$

$R^2 = \frac{4L - 4L^2C\omega^2}{C}$

$R = \sqrt{\frac{4L - 4L^2C\omega^2}{C}}$

$C = ?$

$4L^2C\omega^2 + CR^2 = 4L$

$C(4L^2\omega^2 + R^2) = 4L$

$C = \frac{4L}{4L^2\omega^2 + R^2}$

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MARKOM FS 2020

Bruk av prefikser

Oppgave 13 Prefikser

a) Fyll ut tabellen:

	Forkortes til	som tierpotens	
Terra	T	10^{12}	
Giga	G	10^{9}	1 000 000 000
Mega	M	10^{6}	1 000 000
kilo	k	10^{3}	1000
milli	m	10^{-3}	0,001
mikro	μ	10^{-6}	0,000 001
nano	n	10^{-9}	0,000 000 001
pico	p	10^{-12}	

osv.

b) Hvordan skal EXP-tasten på kalkulatur brukes hvis du ønsker å skrive $3k = 3 \cdot 10^3$?

CASIO: 3 [x10^x] 3 [EXE]

c) Undersøk setup (shift, menu, display) for å se hvilken virkning følgende innstillinger har å si for tallet $3 \cdot 10^3$:
   1) Norm 1
   2) Norm1/E

Oppgave 14 Bruk kalkulator til å regne oppgavene på enklest mulig måte:

a) $154\text{mV} + 1{,}3\text{V} + 950\mu\text{V} =$ (10^{-3}, 10^{-6}) $1{,}45495 \approx 1{,}45\,\text{V}$ (eller $1{,}5\,\text{V}$)

b) $3{,}3\text{A} + 990\mu\text{A} + 56\text{mA} =$ $3{,}356\,\text{A} \approx 3{,}4\,\text{A}$

c) $R = R_1 + R_2 + R_3 = 860\Omega + 1\,\text{M}\Omega + 15\,\text{k}\Omega =$ $1{,}015\,\text{M}\Omega \approx 1{,}0\,\text{M}\Omega$

d) $R = \frac{R_1 \cdot R_2}{R_1 + R_2} = \frac{1\text{k}\Omega \cdot 60\text{k}\Omega}{1\text{k}\Omega + 60\text{k}\Omega} =$ $0{,}983\,\text{k}\Omega \approx 0{,}98\,\text{k}\Omega$

e) $P = \frac{U^2}{R} = \frac{(12\text{V})^2}{15\text{M}\Omega} =$ $9{,}6 \cdot 10^{-6}\,\text{W} = 9{,}6\,\mu\text{W}$